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MATHEMATICAL MODELING OF SALT TRANSPORT BY COUPLED SUBSURFACE AND SURFACE WATER FLOWS

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A mathematical model is examined to describe the transport of salts by coupled flows of surface, soil, and subsoil waters for large-scale objects characterized by complex hydrogeologic conditions. Computational algorithms and computer programs developed for realization of the model are based on the use of finite-difference methods to distinguish between different physical processes and modeling regions. Results are presented from examples of calculations to illustrate characteristic features of the problem of mass transfer by coupled flows.

Introduction. In mathematical modeling of regional water flow and water quality, it is necessary to consider the interaction of different components of a given flow and mass transfer between them. The water flow processes that are part of the hydrologic cycle differ from each other in their physical nature, and each of them has their own region of localization. The following factors make the main contributions to the formation of a water flow [1]: flow in streams and basins; pressurized and unpressurized filtration of subsoil waters in connected aquifers, the migration of moisture in the zone of incomplete saturation (aeration zone), overland runoff of rainwater, and the formation and thawing of the snow cover.

Reliable mathematical models have been developed to describe individual processes in the hydrologic cycle and have long been used to solve practical problems [2, 3]. In the middle of the 1970s, researchers began to actively develop models that describe the coupling of fluvial and filtration flows [4–6]. At the beginning of the 1980s, results began to appear from mathematical modeling of water exchange processes on land with allowance for all relevant factors [1, 7–9]. Such models can be used to predict the regime for large river basins, irrigation and drainage systems, etc. However, the solution of environmental problems requires an evaluation of the quality of subsurface and surface waters.

In this investigation, we propose a mathematical model to describe the transport of salts by coupled flows of surface water (streams and basins), soil water (the aeration zone), and subsoil water for large-scale objects characterized by complex hydrogeologic conditions.

Given the current state of computer technology, it is not yet possible to solve the given problem as a whole on the basis of a single three-dimensional hydrodynamic model. We thus propose to use a modular approach to construct mass-transfer models. The approach involves the linking of hydraulic submodels of varying degrees of complexity which correspond to different physical processes [1]. The interaction between the components of a given water flow are modeled by source functions that enter into differential equations, internal boundary conditions, and parameters of the model determined during the solution of the problem. The overall model of mass transfer includes submodels which describe water exchange and water quality. It is assumed that salt transport has no effect on the flow of the aqueous phase.

Modeling of Coupled Flows of Subsurface and Surface Waters. We are examining a bounded multiply connected modeling region $\Omega \subset R^2$. Inside Ω we make l slits Π_i ($\Pi = \sum_{i=1}^{l} \Pi_i$) corresponding to streams. The closed contours $\Gamma_j = \partial\Omega_{\Gamma j}$ coincide with the fixed boundaries of basins. The internal boundaries

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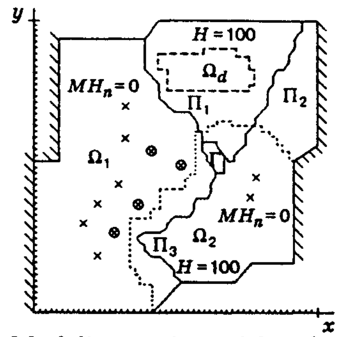

Fig. 1. Modeling region with subregions Ω_i: Π_i are the streams, points $\times$ and $\otimes$ denote the wells pumping water from the top and bottom aquifers.

of Π_i and Γ_j have a point of intersection where the ith stream flows into the jth basin. Figure 1 presents an example of a region with three streams and one basin.

The flow of subsoil water in pressurized and unpressurized aquifers is modeled on the basis of the equations that describe plane filtration [2]

$$\mu H_t = \operatorname{div}(M\nabla H) + r(H_1 - H) + f, \qquad (x,y) \in \Omega, \tag{1}$$

where $H(x,y,t)$ is the level (head) of the subsoil waters, μ and M are coefficients that characterize the specific (elastic) yield of water and water flow, $f(x,y,t)$ is a source function that accounts for the operation of wells and infiltration, and $r(H_1 - H)$ is the flow into the adjacent aquifer with the head H_1. The condition $M = k_f(H - H_b)$ is satisfied for an unpressurized aquifer, where k_f is the filtration coefficient and $H_b(x,y)$ are the upper and lower boundaries of the aquifer.

On the external boundary of the modeling region ($G = \partial\Omega$), we assign boundary conditions of the first and second type

$$MH_n\Big|_{G_1} = \Phi_1, \qquad H\Big|_{G_2} = \Phi_2, \qquad G = \partial\Omega = G_1 \cup G_2. \tag{2}$$

The flow in the drainage Π_i is described by a system of one-dimensional equations for diffusion waves, this system being an approximation of the St. Venant equations

$$\omega_t = (\Psi|z_s|^{1/2}\operatorname{sgn}(z_s))_s - [MH_n]_{\Pi_i} + f_1, \qquad (x,y) \in \Pi_i; \tag{3}$$

$$z\Big|_N = \Phi_N, \qquad \Psi|z_s|^{1/2}\operatorname{sgn}(z_s)\Big|_P = \Phi_P, \qquad z\Big|_Q = u, \tag{4}$$

where $z(s,t)$ is the water level in the stream, $\omega(z,s)$ is the cross-sectional area ($B = \omega_z$ is the width), s is the distance along the stream, $\Psi(z,s) = \eta^{-1}\omega R^{2/3}$ is the discharge modulus, η is the roughness factor ($\eta^{-1} = \gamma$), R is the hydrodynamic radius of the flow, f_1 is a source function, $[MH_n]_\Pi = (MH_n\big|_{\Pi^+} + MH_n\big|_{\Pi^-})$ is the total filtration inflow of subsoil water from the right Π^+ and left Π^- banks of the stream, and $H_n = \partial H/\partial n$ is the outer normal derivative. At the points N, P, and Q (at the beginning and end of the streams), we assign the water level and discharge or we specify that the water levels in the stream and the basin coincide where the stream enters the basin. At a confluence of several streams, the levels of those streams coincide and the inflow equals the outflow.

We assume that the surface waters at the internal boundaries of Π_i are at the same level as the subsoil waters in the first (uppermost) aquifer on the left and right banks:

$$MH_n\Big|_{\Pi^\pm} = \alpha(z - H)\Big|_{\Pi^\pm} \pm \beta(H\Big|_{\Pi^-} - H\Big|_{\Pi^+}). \tag{5}$$

The levels of the subsoil waters on the left and right banks can be assumed to coincide for narrow streams. Then condition (5) changes to the form $q_f \equiv [MH_n]_\Pi = 2\alpha(z - H)\Big|_\Pi$, $H\Big|_{\Pi^+} = H\Big|_{\Pi^-}$.

The change in the water level $u(t)$ in basins with the boundary $\Gamma = \partial\Omega_\Gamma$ are determined by balance relations. Two situations are possible in modeling the interaction of a basin and an upper-lying aquifer. In the first case, within the region Ω_Γ corresponding to the inside part of the basin, the filtration problem is not solved, and the boundary of the basin is the internal boundary of the filtration region. In the second case, the basin is the infiltration region f for the seepage flow calculated inside Ω_Γ. For each type of basin, we assign a balance relation that determines the water level in it, in addition to specifying the compatibility condition for the filtration problem:

$$\lambda u_t = -\oint\limits_\Gamma MH_n ds + q_r + f_2, \qquad MH_n\Big|_\Gamma = \alpha(u-H)\Big|_\Gamma, \qquad (x,y)\in\Gamma; \tag{6}$$

$$\lambda u_t = -\int\limits_{\Omega_\Gamma} \alpha(u-H)dxdy + q_r + f_2, \qquad f = \alpha(u-H), \qquad (x,y)\in\Omega_\Gamma. \tag{6'}$$

Here $\lambda = \operatorname{mes}\Omega_\Gamma$ is the area of the basin, q_r is the flow of water into the basin from the streams that enter it, f_2 is a source function that accounts for evaporation and sedimentation, and f is the source function in (1).

Lower-lying aquifers are not hydraulically coupled with surface streams, and the discharge and head of these aquifers are assumed to be continuous at the internal boundaries of Π.

Vertical migration of soil moisture in the aeration zone is calculated only on the detailed section $\Omega_d \subset \Omega$ and is described by the Richards one-dimensional equation [10]

$$\theta_t = (K(\psi_\eta + 1))_\eta + f_3, \qquad H < \eta < H_p, \qquad (x,y)\in\Omega_d, \tag{7}$$

where $\theta(\psi)$ is the volumetric moisture content, ψ is the pressure of the soil moisture, $K(\eta,\psi)$ is the hydraulic conductivity, and η is the vertical coordinate directed upward. The relations $\theta(\psi)$ and $K(\psi)$ are assumed to exist. For example, it is possible to make use of the empirical formulas $\theta = \theta_s/(1+(-\psi/w)^\sigma)$ and $K = k_s(\theta-\theta_r)^\delta/(\theta_s-\theta_r)^\delta$ for $\psi < 0$ and $\theta = \theta_s = m$ and $K = k_s = k_f$ for $\psi \geqslant 0$.

The absorption of moisture by the root system of the vegetation is accounted for by the source function $f_3 = -E(t)(\theta^*-\theta_0)/(z_k(\theta_k-\theta_0))$ and $\eta\in(H_p-z_k,H_p)$; $f_3 = 0$ and $\eta < H_p - z_k$; $\theta^* = \theta$ and $\theta_0 < \theta < \theta_k$; $\theta^* = \theta_k$ and $\theta \geqslant \theta_k$; $\theta^* = \theta_0$ and $\theta \leqslant \theta_0$.

In this case, on the detailed section Eq. (1) is replaced by the equation

$$\operatorname{div}(M\nabla H) = -K(\psi_\eta+1)\Big|_{\eta=H}, \quad (x,y)\in\Omega_d.$$

The following boundary conditions are assigned at ground level $\eta = H_p$ and at the free surface of the subsoil waters $\eta = H$:

$$K(\psi_\eta+1)\Big|_{\eta=H_p} = R_0(x,y,t), \quad \psi\Big|_{\eta=H} = 0. \tag{8}$$

To close the water problem, we need to assign the initial data

$$H\Big|_{t=0} = H_0, \quad z\Big|_{t=0} = z_0, \quad u\Big|_{t=0} = u_0, \quad \psi\Big|_{t=0} = \psi_0. \tag{9}$$

Modeling the Transport of Contaminating Impurities. Models that describe salt transport by subsurface and surface waters are based on the equations of convective diffusion and account for the exchange of salt between components of the water flow. When salt transport in unpressurized subsoil waters is being modeled, it is necessary to also consider the accumulation of salts in the aeration zone.

Salt transport by a filtration flow in a pressurized aquifer is described by a two-dimensional equation [11]

$$\begin{gathered}(\hat{m}dC)_t = \operatorname{div}(D\nabla C - \mathrm{v}C) - \Phi(C,N) + fC_f^*,\\ \hat{m} = m + \mu\frac{H-H_p}{d}, \quad d = (H_p - H_b), \quad \mathrm{v} = -M\nabla H\end{gathered} \tag{10}$$

with the boundary conditions

$$(D\nabla C - \mathrm{v}C)\mathrm{n}\Big|_{\partial\Omega} = -\mathrm{vn}C_g^*\Big|_{\partial\Omega}, \quad Q_f \equiv [(D\nabla C - \mathrm{v}C)\mathrm{n}]_\Pi = -[\mathrm{vn}C^*]_\Pi. \tag{11}$$

The condition $Q_f = q_f C^*\big|_{\Pi}$ is satisfied for narrow streams.

For a nonconservative impurity, the process of salt deposition on the skeleton of the soil is determined by the ordinary differential equation

$$(dN)_t = \Phi = \gamma(C, N)(C - C_*). \tag{12}$$

Here C and N are the concentrations of salt in solution and in the solid phase, H_p and H_b are the upper and lower boundaries of the aquifer, $C_* = \text{const}$, and $D = D_0 + \lambda|\mathbf{v}|$ is the diffusion coefficient.

The value of C^* is determined by the direction of the filtration flow:

$$C_g^*\big|_{\partial\Omega} = \begin{cases} C, & q < 0, \\ C_g, & q \geqslant 0; \end{cases} \qquad C^*\big|_{\Pi} = \begin{cases} C, & q_f < 0, \\ C_1, & q_f \geqslant 0, \end{cases} \qquad C_f^* = \begin{cases} C, & f < 0, \\ C_f, & f \geqslant 0. \end{cases}$$

Here C_g and C_f are assigned functions and $q = -\mathbf{v}\mathbf{n} = -v_n$.

Equations of the type (10) and (12) are written for each aquifer, and a separate value of concentration is determined in each aquifer. For nonpressurized flow of subsoil waters, the capacity of an aquifer is determined by the relation $d = (H - H_b)$. In a number of cases, the concentration changes only slightly in lower-lying aquifers and can be considered constant.

The transport of impurities by fluvial waters is modeled by a system of one-dimensional equations [12]

$$\frac{\partial}{\partial t}(\omega C_1) = \frac{\partial}{\partial s}\Big(D_1 \frac{\partial C_1}{\partial s} - v_1 C_1\Big) - q_f C^* + f_1 C_1^*, \quad (x, y) \in \Pi_i, \quad v_1 = -\Psi |z_s|^{1/2} \operatorname{sgn}(z_s), \tag{13}$$

where C_1 is the concentration of salts in a stream and $D_1 = \lambda_1 |v_1| R$. At the ends of streams, we assign boundary conditions of the form

$$\Big(D_1 \frac{\partial C_1}{\partial s} - v_1 C_1\Big)\Big|_{N \cup P} = -v_1 C_1^*\big|_{N \cup P}. \tag{14}$$

Salt concentration is averaged over the thickness d of an aquifer and the cross section of streams ω, while for basins it is determined by the balance relations

$$(\lambda(u - u_d)C_2)_t = \oint_{\Gamma} v_n C_2^* \, ds + Q_r + f_2 C_2^*, \quad Q_r = q_r C_2^* \quad \text{for (6)},$$
$$(\lambda(u - u_d)C_2)_t = -\int_{\Omega_\Gamma} \alpha(u - H) C_2^* \, ds + Q_r + f_2 C_2^* \quad \text{for (6')}. \tag{15}$$

Here the values of C_1^* and C_2^* also depend on the flow direction at the corresponding points of the modeling region; u_d is the lower boundary of the basin.

On the detailed section Ω_d, the concentration of a conservative impurity C_3 in the aeration zone is found from the differential equation

$$\frac{\partial(\theta C_3)}{\partial t} = \frac{\partial}{\partial \eta}\Big(D_3 \frac{\partial C_3}{\partial \eta} - v_3 C_3\Big), \quad H < \eta < H_p, \quad (x, y) \in \Omega_d, \qquad v_3 = -K(\psi_\eta + 1). \tag{16}$$

Examining the salt balance in the "vertical column" from the aquifer to ground level, we obtain the following equation to describe salt transport by the filtration flow in the subregion Ω_d

$$\frac{\partial}{\partial t}\Big(\int_H^{H_p} (\theta C_3)\, d\eta + m(H - H_b)C\Big) = \operatorname{div}(D\nabla C - \mathbf{v}C) + f C_f^* + R_0 C_3^*.$$

Having differentiated the first term with respect to time and taking (16) into account, we have

$$m((H - H_b)C)_t = \operatorname{div}(D\nabla C - \mathbf{v}C) + f C_f^* + (-v_3 C_3^* + m H_t C_3)\big|_{\eta = H}. \tag{10'}$$

In the given case, Eq. (10′) is used instead of Eq. (10).

For Eq. (16), we assign the following boundary conditions at ground level and at the free surface of the subsoil waters:

$$\left(D_3 \frac{\partial C_3}{\partial \eta} - v_3 C_3\right)\Big|_{\eta=H_p} = R_0 C_3^*, \qquad \left(D_3 \frac{\partial C_3}{\partial \eta} - v_3 C_3\right)\Big|_{\eta=H} = -v_3 C_3^*\Big|_{\eta=H}. \tag{17}$$

Here, as before, the parameter C_3^* depends on the flow direction.

We need to specify the following initial data for the salt problem:

$$C = C_0, \qquad N = N_0, \qquad C_1 = C_1^0, \qquad C_2 = C_2^0, \qquad C_3 = C_3^0, \qquad t = 0. \tag{18}$$

Computational Algorithm. The numerical realization of the model is based on its decomposition with respect to physical processes and modeling regions [1], which makes it possible to efficiently resolve the problem of dealing with large space-time scales. The salt transport process does not affect the solution of the water problem. Thus, the problem of the quality of the subsurface and surface waters (10)–(18) is separated from the other problems and is solved in the last stage of the investigation, after determination of the velocity field for all of the components of the water flow from the solution of the water exchange problem (1)–(9).

The coupling problem has the following features:

— the type of boundary conditions used for the salt transport problem depends on the direction of the velocity vector of the water flow, which changes over time;

— the equations that describe the flow of the fluvial waters are strongly nonlinear and become degenerate when the stream level coincides with the lower boundary of an aquifer and when sections with a horizontal level appear;

— flows of basin and surface waters are modeled in regions of different dimensions.

The processes accounted for in the model have different characteristic rates, which requires a different degree of detailing of the calculations for each process. For example, the numerical algorithm must permit calculation of the passage of short pulses of an impurity along streams and account for their effect on the salt regime of the subsoil waters. These requirements mean that different time steps must be used in the problems for the fluvial and filtration flows.

We used implicit finite-difference schemes [1, 13] to solve the filtration problem (1) and (2). The algorithm for the calculations is based on the iterative method of variable directions. The following representation was used for the kth time layer to approximate the filtration component of the fluvial flow:

$$q_f^k \equiv [MH_n^k]_\Pi = \alpha(\hat{z}^k - 0.5(H^k + H^{k-1}))\Big|_\Pi, \qquad \hat{z}^k = \tau_f^{-1} \int\limits_{t=t_{k-1}}^{t_k} z\,dt,$$

where $\tau_f = (t_k - t_{k-1})$ is the time step in the filtration problem.

To solve the problem of fluvial flow (3) and (4), we used a modification of the finite-difference scheme proposed in [14]:

$$B\frac{z_i^m - z_i^{m-\delta}}{\tau_p} = \frac{Q_{i,i-1}^m + Q_{i,i+1}^m}{\Delta s} + f_i^m + q_f^m, \quad \delta = \frac{1}{K},$$

$$Q_{i,j}^m = Q_{i,j}^{m-\delta} + \frac{\partial Q_{i,j}}{\partial z_i}\Big|_{z^{m-\delta}}(z_i^m - z_i^{m-\delta}) + \frac{\partial Q_{i,j}}{\partial z_j}\Big|_{z^{m-\delta}}(z_j^m - z_j^{m-\delta}),$$

$$\frac{\partial Q_{i,j}}{\partial z_k} = 0.5\left(\frac{\partial \psi_k}{\partial z_k}\frac{|z_j - z_i|^{1/2}}{(\Delta s)^{1/2}}\operatorname{sgn}(z_j - z_i) + \frac{\delta_k(\psi_i + \psi_j)}{2(|z_j - z_i|^{1/2} + \varepsilon)}\right), \qquad k = i, j.$$

Here τ_p is the time step for the fluvial flow problem, which is smaller that the step τ_f for the filtration problem $\tau_f/\tau_p = K \geqslant l \geqslant 1$, $m = n + l/K$, $q_f^m = 2\alpha(z^m - \hat{H})$ is the filtration inflow, $\delta_j = 1$ and $\delta_i = -1$, $\hat{H} = (l/K)H^{n+1} + (1 - l/K)H^n$, and Δs is the step for the space variable, reckoned along the channel of the stream. Introduction of the regularizing parameter $\varepsilon > 0$ makes it possible to calculate the flow in streams in the presence of isolated points with a horizontal water level.

Implicit conservative finite-difference methods were also used to solve the salt problem in the subsoil waters. A local salt balance referred to the corresponding elementary rectangle of the grid is assigned to each node. The convective component was approximated by using a countercurrent scheme, and the direction of flow of the aqueous phase was considered in determining the contribution of exchange processes to the salt balance. The rate of transfer of salts Q_{ij} from the ith component of the water flow to the jth component of the same flow is determined by the relation $Q_{ij} = q_{ij}C^*_{ij}$, where $C^*_{ij} = \{C_i, q_{ij} > 0;\ C_j, q_{ij} \leqslant 0\}$ (q_{ij} is the rate of flow of water from the ith component to the jth component).

Examples of Calculations. We shall present numerical calculations for two problems concerning the transport of a contaminating impurity by interacting fluvial and filtration flows. The filtration region ($\Omega = \Omega_1 \cup \Omega_2$) for both problems (Fig. 1) is contained within a rectangle $\{0 < x < 12{,}000$ m, $0 < y < 17{,}000$ m$\}$. The drainage consists of three streams and a basin, all of which are hydraulically coupled with an upper-lying aquifer. The detailed section Ω_d is taken into account only in the second problem.

In the *first problem*, filtration of subsoil waters occurs in two hydraulically coupled pressurized aquifers. The filtration parameters in regions Ω_1 and Ω_2 were assigned the following values:
— for the upper-level aquifer, $M_1 = 600$ m^2/day, $m_1 = 0.22$, and $\mu_1 = 0.001$, $M_2 = 800$ m^2/day, $m_2 = 0.17$, and $\mu_2 = 0.001$, and $d_1 = d_2 = 30$ m;
— for the lower-level aquifer, $M_1 = 650$ m^2/day and $\mu_1 = 0.05$, $M_2 = 400$ m^2/day and $\mu_2 = 0.04$.
The coefficient r in the formula for flow between the aquifers is equal to 0.0001 m/day.

The parameters characterizing the stream channels have the following values: $B_1 = 3$ m, $\alpha_1 = 2.2$ m/day, and $\gamma_1 = 1.5 \cdot 10^6$, $B_2 = 2$ m, $\alpha_1 = 2.0$ m/day, and $\gamma_2 = 1.6 \cdot 10^6$, and $B_3 = 4$ m, $\alpha_1 = 2.6$ m/day, and $\gamma_3 = 1.8 \cdot 10^6$.

The parameters of the basin are as follows: $\lambda = 9.0 \cdot 10^5$ m^2, $\alpha = 0.15$ m/day, and the lower boundary $u_d = 95$ m. The water level was determined from the balance relation (6′), where q_r accounts for the flow of water into the basin via the two streams Π_1 and Π_2, and the flow of water out of the basin via the third stream Π_3.

The boundary conditions adopted for the water and salt problems at the boundary of the filtration region G are constant over time, while the conditions at the inlet ($s = 0$) of streams Π_1 and Π_2 change over a period of 365 days in accordance with the following law: $z = 101 + \delta_i(100 - t)$ and $t \leqslant 100$ days, $z = z_i - \delta_i(200 - t)$ and $100 < t \leqslant 200$ days, $z = 101$, $200 < t < 365$ days, and $s = 0$, $\delta_1 = 0.03$ m/day and $z_1 = 104$ m, $\delta_2 = 0.025$ m/day and $z_2 = 103.5$ m.

A solution with a concentration of 10 mg/liter entered streams Π_1 and Π_2 at the beginning of the modeled period $0 \leqslant t < 200$ days. For the remainder of the period, $200 \leqslant t < 365$ days, the concentration of the solution was zero. A constant level $z = 99.5$ m was assigned at the outlet of the third stream.

The initial distribution of subsoil waters and fluvial waters corresponds to a periodic steady-state regime (at the beginning of the period) with the assumption that no wells are in operation. The initial value of concentration throughout the region is zero. In the second aquifer, we assumed that the concentration was zero for the entire time of calculation. The parameters $D_0 = 10$, $\lambda = 2.5$, and $\lambda_1 = 0$ determine the coefficient of diffusion of salts in subsoil waters and streams.

The yield of the wells Q_C that pump water from the lower-lying (second) aquifer is 3000 m^3/day. The yield of the wells which pump water from the upper-lying aquifer is 2000 m^3/day.

The problem was solved on a difference grid with 51×47 nodes. The steps of the space variables were changed within the following ranges: $\Delta x = 200$–300 m and $\Delta y = 300$–500 m. The time step is $\tau_f = 10$ days in the filtration problem and $\tau_r = \tau_f/100$ for the drainage.

Figures 2 and 3 show the results of calculation of the filtration flow and salt transport in subsoil waters. The regime of the subsoil waters becomes nearly periodic over 10 theoretical years, and a band of salt contamination is formed in these waters and expands slowly over time. Figure 2 shows the contours of the water tables in the upper and lower aquifers ($T = 7300$ days). Figure 3 shows isolines of salt concentration in the upper aquifer for the theoretical time $T = 10{,}950$ days.

Salts enter subsoil waters only as a result of filtration of the solution from streams and basins. The contamination is then slowly carried in the direction of pumping wells by filtration flow within the given

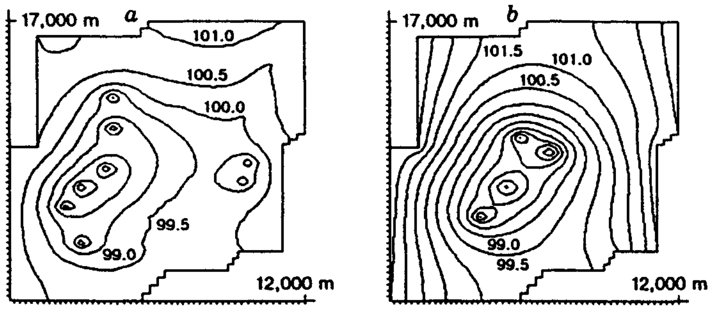

Fig. 2

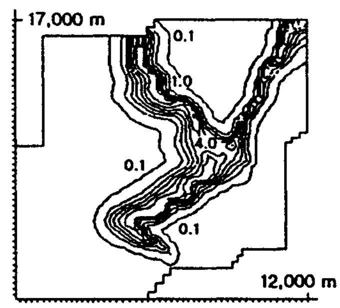

Fig. 3

Fig. 2. Water-table contours in the upper (a) and lower (b) aquifers.

Fig. 3. Isolines of salt concentrations in the upper aquifer.

region. When salts stop entering streams Π_1 and Π_2, their concentration in the streams decreases to zero over a period of several days. Despite this, salt concentration remains nontrivial in the basin and the third stream for an entire year and increases to nearly 10 mg/liter at the beginning of the period. Concentration decreases after salt stops entering the streams. The lowest salt concentration in the basin is reached at the end of the period. Values of 0.373, 0.409, and 0.426 mg/liter were obtained for salt concentration in the basin over the periods 3650, 7300, and 10,950 days, respectively. The accumulation of salts is due to the storage properties of the basin and depends on the rate at which the water in the basin is replenished.

In the second problem, filtration of the subsoil waters occurs in one unpressurized aquifer with the parameters $m_1 = 0.2$, $k_f^1 = 8$ m/day, $m_2 = 0.15$, and $k_f^2 = 10$ m/day, where m_i and k_f^i are the porosity and the filtration coefficient in the region Ω_i ($i = 1, 2$). The aquifer and the surface of the ground are horizontal ($H_b = 75$ m and $H_p = 110$ m). On the detailed section Ω_d, we use Eq. (7) to calculate the migration of moisture in the aeration zone (see Fig. 1). The water flow regime is steady with constant boundary conditions for the fluvial and filtration flows and constant discharges in wells with a yield $Q_C = 2000$ m^3/day. A constant level $z\big|_{s=0} = 101$ m was assigned at the inlet of streams Π_1 and Π_2, while the remaining parameters of the streams coincide with the values in the first problem.

For the aeration zone, the flow rate was assigned on the ground surface ($R_0 = 0.002$) and the absorption of moisture by plants' root was taken into account ($E = 0.002$). The soil parameters ($\theta_1 = 0.02$, $\theta_2 = 0.2$, $w = 5$ m, $\delta = \sigma = 3$, and $k_f = 8$ m/day) and root-system parameters ($\theta_k = 0.2$, $\theta_z = 0.05$, and $z_k = 1$ m) were given. On the detailed section, the flow from the aeration zone is infiltration supply to subsoil waters, and it is determined in solving the stationary problem of water exchange. A solution with a concentration of 10 mg/liter is continuously supplied from the aeration zone, and there are no other sources of concentration in the modeling zone.

A contamination spot that expands over time is formed in the subsoil waters in and near the subregion Ω_d. After 5 years, the contamination reaches streams Π_1 and Π_2. Since these streams drain subsoil waters, the salts enter the streams and, moving through the drainage, also enter the basin and the stream Π_3. Conversely, water filters into the soil from stream Π_3, which leads to contamination of the subsoil waters near that stream. The concentration of salts in the basin slowly increases over time, with the values 0.081, 0.205, and 0.313 mg/liter that we calculated corresponding to times of 3650, 7300, and 10,950 days, respectively. Although streams Π_1 and Π_2 drain subsoil waters, they cannot prevent spreading of the contamination from one bank to the other by the filtration flow. Figure 4 shows results of calculation of the filtration flow and salt transport in the subsoil waters ($t = 10,950$).

Conclusion. The examples examined above illustrate several characteristic features of salt transport by coupled flows. Due to the low rate of flow of subsoil waters, salts accumulate in them and local contamination spots are formed. The filtration flow from the drainage in the first example and infiltration from the aeration

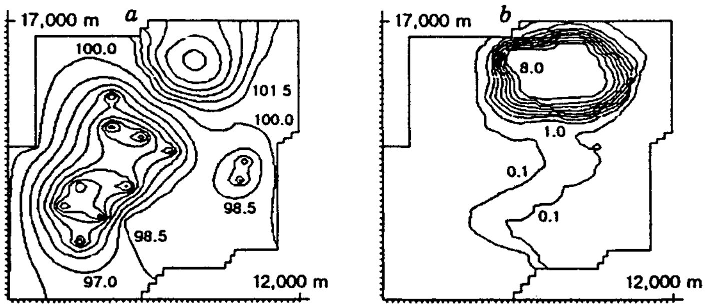

Fig. 4. Water-table contours (a) and isolines of salt concentration (b) in subsoil waters.

zone in the second example are the main factors that determine the subsoil-water supply region. In such cases, even the entry of contaminants into the filtration flow at a low rate will lead to a significant increase in their concentration near that region. The salt solutions are carried at a high rate by streams and in a short period of time salinate subsoil waters located large distances from the original source of contamination.

The above model of water exchange can be used to construct hydrologic models of any degree of complexity for ongoing practical use. The model can also be used to make real-time predictions of water and salt conditions in subsoil and surface waters within limited areas. Such predictions can then be used to optimize the regime of operation of hydrological facilities (irrigation and drainage systems, intakes, canals, reservoirs, etc.), evaluate the environmental losses from their operation, and plan measures to compensate for the adverse effects on the environment.

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